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# Interaction of Swept and Unswept Normal Shock Waves with Boundary Layers

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**The results of an attempt to correlate some of the flow features observed in the interaction between a turbulent boundary layer on a wall and a swept shock wave generated by a fin normal to the wall with corresponding features in the interaction between a turbulent boundary layer and an unswept normal shock are presented. It is found that the Mach number normal to the front shock in both swept and unswept interactions can be correlated in terms of the Mach number normal to the main shock. Given this correlation it can be shown that certain angles measured from flow-visualization photographs are not functions of the Mach number normal to the front shock alone. This shows why previous workers have found correlations for certain geometric parameters in swept interactions but not for others.**

## Introduction

IN a recent series of papers, Settles and Dolling have made an extensive study of the interaction between a turbulent boundary layer on a wall and a swept shock wave generated by a fin normal to the wall.[1] Their results show that outside an inception zone the flow is virtual conical,[1] and Alvi and Settles have produced striking conical shadowgraph[2] and vapor screen photographs[3] of the shock structure in the plane normal to the axis of the conical flow. They have also shown that some of the features of the conical flow can be correlated in terms of the Mach number normal to the main fin shock ($M_n = M_\infty \sin\beta_s$, from Fig. 1), whereas other features do not appear to correlate in terms of this parameter. In another series of papers,[4,5] the present author and his co-workers have reported on various aspects of the interaction between a turbulent boundary layer and an unswept normal shock. For these flows $M_\infty \equiv M_n$. This paper presents the results of an attempt to correlate shock angles and pressure ratios in the swept and unswept flows in terms of $M_n$ and hence to shed more light on the significance of normal Mach number in the swept interaction. (Most of the normal shock measurements have been made in the Mach number range $1.2 < M_\infty < 1.6$, since this is the main range of interest in wing design and in bypass fans, whereas the range of $M_n$ in the swept interactions is $1.2 < M_n < 2.5$, covering the range of interest in engine intakes. This paper includes a new photograph of the normal interaction at $M_\infty \equiv M_n = 1.8$.)

## Correlation of Swept and Unswept Interactions

Figure 2 shows a spark schlieren photograph of the two-dimensional interaction at $M_\infty = 1.8$, obtained using the technique described by Atkin and Squire.[4] A clear lambda shock can be seen above the separated flow under the interaction, and the slip line is particularly distinct downstream of the triple point. Overall the shock system at this Mach number is very similar to planar laser scattering (or vapor screen) prints shown in Figs. 9 and 10 of Alvi and Settles for similar values of $M_n$ (1.82 and 1.88) but completely different freestream Mach numbers (2.95 and 3.95), thus suggesting that $M_n$ is the main parameter fixing the flow pattern. Differences in the flow pattern must be expected downstream of the shock and in the separated region. The interaction region becomes more swept with increase in freestream Mach number so that the axial component of velocity along the core of the separated flow increases, leading to a stronger vortex flow. In the case of the unswept interaction, the separated flow does not form a vortex, and reattachment which is controlled by the pressure gradient downstream of the interaction, is particularly sensitive to the tunnel configuration. The formation of a vortex flow in the swept case does not preclude a successful correlation with the unswept interaction since a similar correlation has been found for the condition for the onset of leading-edge separation on swept wings at transonic and supersonic speeds. For swept wings, leading-edge separation results in a strong vortex flow inboard of the edge, and it has been shown[6] that the conditions for onset of separation correlate in terms of Mach number ($M_N$) and incidence ($\alpha_N$) normal to the edge. [For a swept wing, $M_N = M_\infty \cos\lambda(1 + \sin^2\alpha \tan^2\lambda)$ and $\alpha_N = \tan^{-1}(\tan\alpha \sec\lambda)$, where $\alpha$ is the wing incidence and $\lambda$ the sweepback angle of the leading edge.] Furthermore it was shown that this boundary between separated and nonseparated flow at the leading edge for swept wings in the $M_N$–$\alpha_N$ plane was almost identical to that found for the corresponding boundary[7] for leading-edge separation on unswept aerofoils in terms of $M_\infty$ and $\alpha$. Later work[8,9] in this area has shown that many other features in the flow over swept wings, including the formation of conical shocks above the separated vortex, reattachment, and inboard shock-induced separation, can be correlated in terms of $M_N$ and $\alpha_N$; however, there are no corresponding features in the unswept flow, and so complete correlation is not possible.

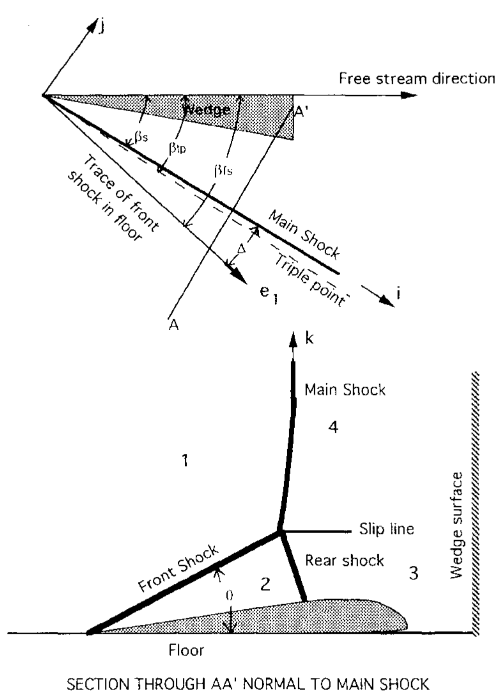


**Fig. 1 Sketch of swept shock pattern and coordinate system.**

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As pointed out by Settles, complete correlation cannot be expected downstream of the triple point in the swept shock case since for a given wedge angle the oblique shock moves closer to the wedge surface with increase in freestream Mach number and the flow pattern downstream of the triple point is essentially confined to a smaller angular region.

In the case of the slip line there appears to be an obvious difference between the swept and unswept interaction since the slip line is deflected away from the surface in the unswept interaction and appears to be deflected towards the surface in the swept case. This difference is a result of the method of observation. In the normal, or unswept, interaction the slip line seen in Fig. 2 indicates the true deflection of the flow through the triple point that, for $M_\infty = 1.5$ and 1.8, is away from the surface. For the swept interaction the line seen in the vapor screen print is the trace of the conical slip surface passing through the laser light sheet (the laser light sheet is normal to the main shock; i.e., it is parallel to AA′ in Fig. 1). Such a surface could appear to slope towards the test surface even if the velocity downstream of the triple point has a component of velocity away from the surface. In fact, all of the flow photographs presented by Alvi and Settles show that the main shock immediately above the triple point is slightly swept back. Calculations based on a simple extension of the shock analysis outlined in Eqs. (1–4) show that the flow that has passed through the main shock just above the triple point does have a velocity component away from the surface.

Correlation in terms of $M_n$ might be expected ahead of the triple point. However, even a cursory comparison of Fig. 2 with the corresponding photographs for the swept interaction (Figs. 5–11 of Ref. 2) suggests that the inclination of the front shock $\theta$ (see Fig. 1) is greater in two-dimensional flow than in the swept shock cases. This impression is confirmed by the results plotted in Fig. 3, where values of $\theta$ for the swept interaction (measured from published figures in papers by Alvi and Settles[2,3]) are compared with the corresponding values of $\theta$ for the unswept interaction taken from schlieren and shadowgraph photographs published by Atkin and Squire,[4] Seddon,[10] and East et al.[11] From this figure it can be seen that the two-dimensional results are larger than the values of $\theta$ for the swept interaction. For the swept interaction, $\theta$ appears to correlate with $M_n$, and a similar collapse was shown by Alvi and Settles in Fig. 18 of Ref. 2 based on much of the same data used here. However, in two-dimensional flow, $\theta$ represents the total inclination of the front shock, whereas in the swept interaction the conical front shock has an additional inclination out of the crossflow plane. A measure of the total inclination of the front shock on the flow is provided by the component of the freestream Mach number normal to it. This then completely fixes the flow in region 2 downstream of the front shock (see Fig. 1). To calculate this normal Mach number in terms of the conical geometry it is convenient to set up a system of orthogonal unit vectors normal to (and in the plane of) the main shock centered at the origin of the virtual conical flow. (As pointed out by Alvi and Settles, the origin of the virtual conical flow does not lie at the vertex of the wedge but slightly ahead of it. For the present calculation the origin is taken as the intersection of the leading edge of the wedge with the test surface. This introduces a slight error in the calculated value of $M_{nf}$, but sample calculations show that this error is far smaller than that introduced by reading angles from flow visualization pictures.) In this system, $\boldsymbol{k}$ is a unit vector normal to the test surface, $\boldsymbol{i}$ lies along the main shock, and $\boldsymbol{j}$ completes the right-handed system. The unit vector normal to the plane of the front shock $\boldsymbol{n}$ can be found from the crossproduct of the vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, where $\boldsymbol{e}_1$ is a unit vector lying through the intersection of the plane of the front shock with the floor and $\boldsymbol{e}_2$ is a unit vector lying through the intersection of the extension of the plane of the front shock with the main shock. From Fig. 1 it can be seen that

$$\boldsymbol{e}_1 = \cos\Delta \boldsymbol{i} - \sin\Delta \boldsymbol{j}, \qquad \text{where} \qquad \Delta = (\beta_{fs} - \beta_s) \tag{1}$$

$$\boldsymbol{e}_2 = \cos\phi \boldsymbol{i} + \sin\phi \boldsymbol{k}, \qquad \text{where} \qquad \phi = \tan^{-1}\{\tan\theta \tan\Delta\} \tag{2}$$

In the same coordinate system the freestream Mach number is

$$\boldsymbol{M}_\infty = M_\infty \cos\beta_s \boldsymbol{i} + M_\infty \sin\beta_s \boldsymbol{j} = \sqrt{M_\infty^2 - M_n^2}\boldsymbol{i} + M_n \boldsymbol{j} \tag{3}$$

where $M_n$ is the magnitude of the component of the freestream Mach number normal to the main shock. The Mach number normal to the front shock, $\boldsymbol{M}_\infty \cdot \boldsymbol{n}$, is thus given by

$$\frac{\sqrt{M_\infty^2 - M_n^2}\sin\Delta\sin\phi + M_n \sin\phi\cos\Delta}{\sqrt{\sin^2\Delta\sin^2\phi + \cos^2\Delta\sin^2\phi + \sin^2\Delta\cos^2\phi}} \qquad (= M_{nf}, \text{say}) \tag{4}$$

The pressure ratio $(p_2/p_1)$ and the temperature ratio $(T_2/T_1)$ across the front shock, together with the normal Mach number downstream of the shock, can then be found from $M_{nf}$ using the normal shock relations, and the rest of the flow properties immediately downstream of the front shock follow. Values of $\Delta$

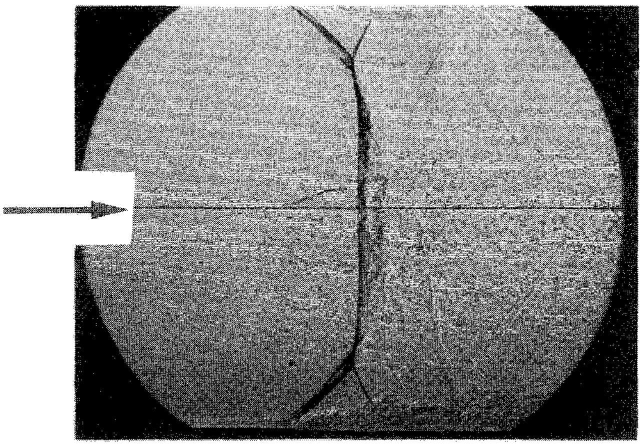

Fig. 2 Spark shadowgraph of normal shock interaction in two-dimensional flow at $M_\infty$ = 1.8.

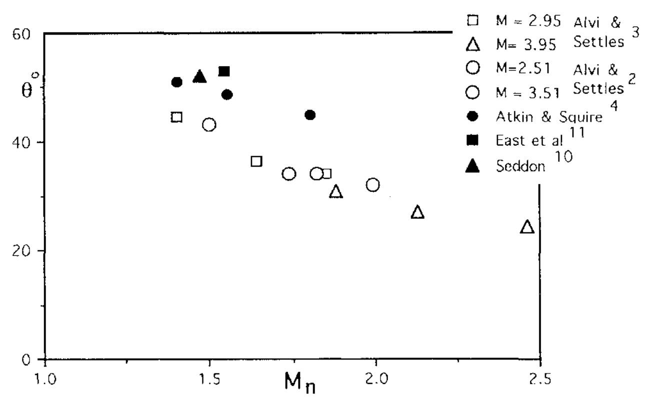


**Fig. 3 Variation of $\theta$ with normal Mach number: swept (open symbols) and unswept (solid symbols) interactions.**

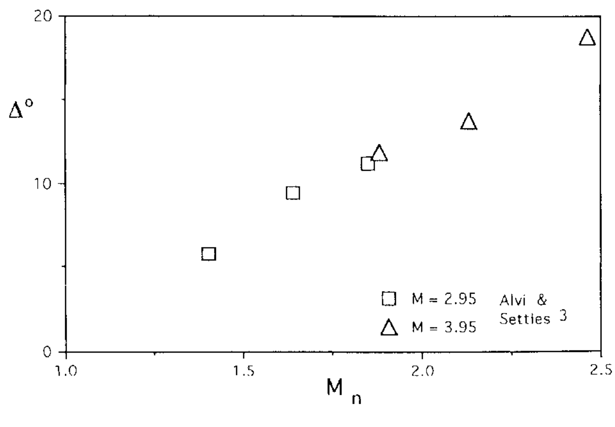


**Fig. 4 Variation of $\Delta$ [= $(\beta_{fs} - \beta_s)$] for front shock with normal Mach number for swept interactions.**

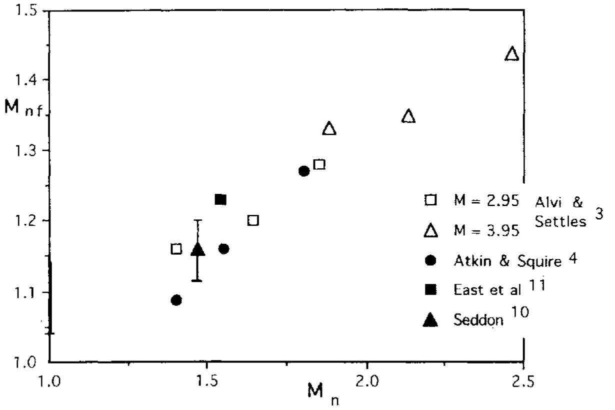


**Fig. 5 Variation of Mach number normal to front shock with normal Mach number: swept (open symbols) and unswept (solid symbols) interactions.**

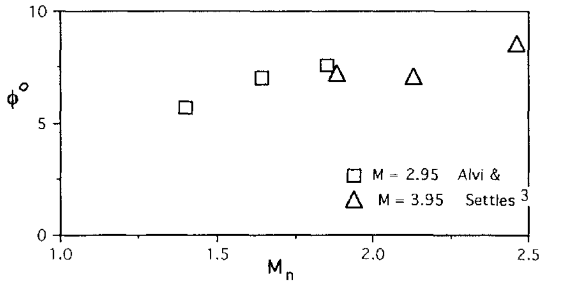


**Fig. 6 Variation of $\phi$ with normal Mach number: swept interactions.**

are plotted against $M_n$ in Fig. 4. These values are taken from Alvi and Settles,[3] and as shown by Settles[12] and Alvi,[13] the results for $M_\infty = 2.95$ and 3.95 appear to collapse when plotted against $M_n$. When $M_{nf}$, as calculated from Eq. (4), is plotted against $M_n$ (Fig. 5) for both the swept and unswept interactions, all of the points collapse within the estimated scatter. (Values of $\theta$ and $\Delta$ are read from published graphs and are estimated to have an accuracy of $\pm 2$ deg, leading to a possible error of $\pm 0.04$ in $M_n$.)

If this apparent collapse of $M_{nf}$ with $M_n$ is genuine, then both $\Delta$ and $\theta$ cannot be unique functions of $M_n$ since if they were, then so would $\phi$ [see Eq. (2)], and hence by Eq. (4) $M_{nf}$ would be a function of $M_n$ and of $\sqrt{(M_\infty^2 - M_n^2)}$. In fact, direct calculation of $\phi$ from the results of Figs. 3 and 4 shows that $\phi$ does not correlate with $M_n$ (Fig. 6). Since $\tan\phi$ is directly related to the height of the triple point, this shows that this height also does not correlate with $M_n$ as is confirmed by the results plotted in Fig. 22 of Alvi and Settles.[14]

## Conclusions

The present results suggest that correlations between swept and unswept normal shock interactions can be made in terms of the $M_n$, the Mach number normal to the main shock, provided the correlations are made in terms of local Mach numbers and pressure ratios. Taken in conjunction with the analysis of Eqs. (1–4), this shows that angles and heights measured from flow visualization photographs are not always functions of $M_n$ only. This shows why previous workers have found correlations for certain geometric parameters in swept interactions but not for others. Finally, note that conditions at the triple point in two-dimensional flow were studied in considerable detail by Henderson in a series of papers.[15,16] Based on this work, McGregor[17] produced a table giving the angle of the main shock, the rear shock, and the slip line in terms of the deflection through the front shock and of $M_\infty$ ($1.3 < M_\infty < 4$) for a two-dimensional triple point. Measurements of the various shock angles for two-dimensional flows are, as expected, in close agreement with values given in McGregor's table. Clearly the analysis of Eqs. (1–4) could be extended to study the flow at the conical triple point. The exact shock angles so obtained would provide a framework for improved numerical solutions in this region and might provide further correlations between the swept and unswept interactions.

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